Generalization of Luttinger's Theorem for Strongly Correlated Electron Systems

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Analyzing general structure of the Green function of a strongly correlated electron system we have shown that for the regime of strong correlations Luttinger's theorem should be generalized in the following way: the volume of the Fermi surface of the system of noninteracting particles is equal to volume of the Fermi surface of the quasiparticles in the strongly correlated system with account for the spectral weight of these quasiparticles. Hubbard and t-J models analysis in the paramagnetic nonsuperconducting phase shows that the generalized Luttinger's theorem is valid for these models.

I. INTRODUCTION

At present, it is widely believed that Luttinger's theorem [1] is violated for strongly correlated electron systems (SCES), among which is, in particular, the broad class of superconducting cuprates. This theorem states that the volume of the Fermi surface of interacting particles is equal to that of the noninteracting particles. The proof of Luttinger's theorem [1] is valid for the normal Fermi liquid only. A topological non-perturbative proof of this theorem for SCES was given in [2] on the assumption that these systems are normal Fermi liquids. Since this proof is based on general considerations it is valid for the t-J model and the Hubbard model in the Fermi liquid phase. However in the SCES other phases are also exist, whose properties differ from those of the Fermi liquid. Deviations from the Fermi liquid behavior reveal themselves in the redistribution of the spectral weight of a quasiparticle between different Hubbard subbands and in the fact that the imaginary part of the self-energy $\text{Im}\Sigma_k(E)$ is nonzero on the Fermi surface. Indeed, the relationship $\text{Im}\Sigma_k(E) \propto (E - \varepsilon_F)^2$ is valid in the vicinity of the Fermi level ε_F of the Fermi liquid.

Calculations of the Hubbard model in the framework of the dynamical mean field theory (DMFT), which is exact in the limit of infinite dimensionality ($d = \infty$) [3, 4, 5], demonstrated that the distribution function of the quasiparticles has a jump in the vicinity of the Fermi level. This jump continuously decreases down to zero with increasing the parameter of the on-site Coulomb repulsion U. Nevertheless, the Fermi liquid picture persist up to a certain critical value U_c in this case, after which the system transfers to the insulating state. Edwards and Hertz [6], however, using an interpolation scheme for the Hubbard model not based on the limit $d = \infty$ demonstrated that the imaginary part of the self-energy is nonzero on the Fermi surface at $U \propto U_c$ and near the half-filling ($x \ll 1$, where $n = 1 - x$ is the electron concentration). But with a deviation from the half-filling, the Fermi liquid properties of the system are restored

fairly rapidly. That the Fermi liquid properties in the Hubbard model with $U = 8t$ (t is the hopping integral) begin to be restored already at $x > 0.1$ was shown in [7] in the framework of the non-perturbative approach of dynamical cluster approximation (DCA). At present, the problem of the transition from the Fermi liquid phase to a metallic non-Fermi liquid state with strong electron correlations and the behavior of the system in the transition region has only been stated and is far from being solved (see e.g. [8]).

Since in real materials the transition to the metal state is observed at $x \ll 1$, it is the most interesting range of concentrations. At extremely small values of x it would be expected that the additional carriers will be localized in the vicinity of the bottom of the band and the condition $\text{Im}\Sigma_k(\varepsilon_F) \neq 0$ will be valid for them. As x increases further the Fermi level falls within the range of delocalized states, for which the imaginary part of the self-energy is equal to zero but the non-Fermi liquid effects are still present due to redistribution of the spectral weight between the Hubbard subbands. Deviations from Luttinger's theorem for the Hubbard model in the region where $\text{Im}\Sigma_k(\varepsilon_F) \neq 0$ were discussed in [9] in the framework of the FLEX approximation. In this paper, we restrict ourselves to the concentration range with $\text{Im}\Sigma_k(\varepsilon_F) = 0$.

As for the case when the spectral weight of a quasiparticle in SCES is not equal to unity, the properties of the system are different from those of the normal Fermi liquid and the original Luttinger theorem is violated. Indeed, the Fermi momentum of the Hubbard bands is larger than the Fermi momentum for free electrons at the same Fermi energy (Fig. 1), so the geometric volume of the Fermi surface is larger for the Hubbard bands [10]. However, in this case the system is in the metallic state and since the distribution function of quasiparticles undergoes a jump in the vicinity of the Fermi level, Luttingers theorem can be generalized to quasiparticles in the following way: the volume of the Fermi surface of noninteracting particles is equal to that of the interacting quasiparticles with account for the spectral weight of these quasiparticles. In the present work, by analyzing the general structure of the Green function and thorough investigation of the Hubbard-I solution of the t-J mode

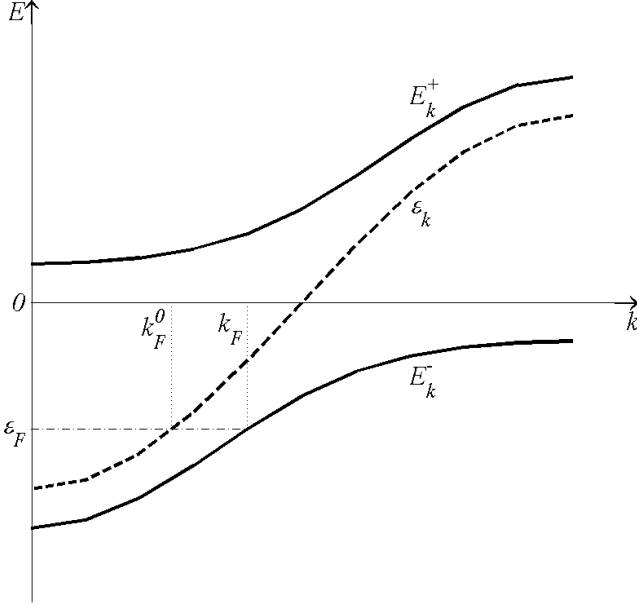*Electronic address: mkor@iph.krasn.ru

FIG. 1: Dispersion curves of the Hubbard bands $E_k^{\pm}$ and the one-particle spectrum ε_k. ε_F is the Fermi level, k_F and k_F^0 are the Fermi momenta for the Hubbard bands and free electrons respectively.

and Hubbard model [11], we shown that this generalized formulation of Luttingers theorem is valid for metallic SCES. Actually, such a metallic system is not rather a normal but a "compressible" Fermi liquid, which is due to the spectral weight of the quasiparticles being different from unity and to the Fermi surface being less compact (less "dense"). This ideas makes it possible to eliminate the inconsistency between the concentration of the excess carriers in the superconducting cuprates and the unduly large volume of the Fermi surface calculated in the framework of the model of the normal Fermi liquid.

II. ANALYSIS OF THE GREEN FUNCTIONS' GENERAL STRUCTURE

Luttinger has shown [1] that the equality between the volumes of the Fermi surface in the momentum space for interacting particles V_{FS}, and for noninteracting particles V_{FS}^0,

$$V_{FS} = V_{FS}^0, \qquad (1)$$

follows from the fact that the average number of particles $\langle N \rangle$ for interacting and noninteracting fermions is the same. Indeed, for a system without interaction, we have

$$
\begin{aligned}
\langle N \rangle &= \sum_k \theta\left(\mu_0 - \varepsilon_k\right) \\
&= \frac{V}{(2\pi)^3} \int dk\, \theta\left(\mu_0 - \varepsilon_k\right) = \frac{V}{(2\pi)^3} V_{FS}^0. \quad (2)
\end{aligned}
$$

and, for the Fermi liquid system with interaction [1], we have

$$
\begin{aligned}
\langle N \rangle &= \sum_k \theta\left(\mu - \varepsilon_k - \mathrm{Re}\Sigma_k\right) \\
&= \frac{V}{(2\pi)^3} \int dk\, \theta\left(\mu - \varepsilon_k - \mathrm{Re}\Sigma_k\right) = \frac{V}{(2\pi)^3} V_{FS}(3)
\end{aligned}
$$

Here, V is the volume of the system of fermions; μ and μ_0 are the chemical potentials of the system with and without interaction, respectively; ε_k are one-particle energies; $\mathrm{Re}\Sigma_k$ is the self-energy part of the Green function; and $\theta\left(x\right)$ is the Heaviside unit-step function.

For strongly correlated electron systems, however, the definition of the average number of particles as the sum of the Heaviside unit-step functions is invalid, because the spectral weight of each quasiparticle in the system is taken to be the unit in this definition. One of the essential peculiarities of strongly correlated electron systems is a variation of the spectral weight from unity in each band due to its redistribution between the Hubbard subbands at $U \gg W$ (W is the half-width of the band). For this reason, analogs of Eqs. (2) and (3) should be derived for this case.

In what follows, we use the Hubbard X-operators [12] defined in the following way: $X_f^{pq} \equiv |p\rangle \langle q|$, where $|p\rangle$ and $|q\rangle$ are states at site f. Since the number of root vectors $\alpha\left(p,q\right)$ is finite, they can be enumerated; thus, we have

$$X_f^{pq} \leftrightarrow X_f^{\alpha(p,q)} \leftrightarrow X_f^{\alpha_m} \leftrightarrow X_f^m. \qquad (4)$$

Here, index $m \leftrightarrow (p,q)$ enumerates quasiparticles with the energies

$$\omega_m = \omega_{pq} = \varepsilon_p\left(N+1\right) - \varepsilon_q\left(N\right), \qquad (5)$$

where ε_p is the energy level with index p for the N-electron system.

The Hubbard operators are related to one-electron creation and annihilation operators in the following way:

$$a_{f\lambda\sigma}^+ = \sum_m \gamma_{\lambda\sigma}^*\left(m\right) X_{f\sigma}^m{}^+, \quad a_{f\lambda\sigma} = \sum_m \gamma_{\lambda\sigma}\left(m\right) X_{f\sigma}^m, \quad (6)$$

where $\gamma_{\lambda\sigma}\left(m\right)$ is the partial weight of a quasiparticle m with spin σ and orbital index λ.

The average occupation numbers $\langle n_{k\lambda\sigma}\rangle$ for the particles with momentum k and spin σ are expressed in terms of the electron Green function written in the energy representation, $G_{k\lambda\sigma} = \langle\langle a_{k\lambda\sigma} \mid a_{k\lambda\sigma}^+ \rangle\rangle_{E+i\eta}$, in the following way:

$$\langle n_{k\lambda\sigma}\rangle = \int dE\, f_F\left(E\right)\left(-\frac{1}{\pi}\mathrm{Im}G_{k\lambda\sigma}\right), \qquad (7)$$

where $f_F(E)$ is the Fermi function, $\eta \to 0$, $\eta > 0$. In the X-operators representation, the Green function has the form

$$
\begin{aligned}
&\langle\langle a_{k\lambda\sigma} \mid a_{k\lambda\sigma}^+ \rangle\rangle_{E+i\eta} \\
&= \sum_{m,p} \gamma_{\lambda\sigma}\left(m\right)\gamma_{\lambda\sigma}^*\left(p\right)\left\langle\left\langle X_{k\sigma}^m \mid X_{k\sigma}^p{}^+ \right\rangle\right\rangle_{E+i\eta}. \quad (8)
\end{aligned}
$$

For the matrix Green function $D_{k\sigma}^{m,p}(E) = \left\langle\left\langle X_{k\sigma}^m \Big| X_{k\sigma}^{p^+} \right\rangle\right\rangle_{E+i\eta}$, the generalized Dyson equation [13] can be written as

$$\hat{D}_{k\sigma}(E) = \left\{ \left[\hat{G}_{k\sigma}^{(0)}(E)\right]^{-1} + \hat{\Sigma}_{k\sigma}(E) \right\}^{-1} \hat{P}_{k\sigma}(E). \quad (9)$$

Here, $\hat{\Sigma}_{k\sigma}(E)$ and $\hat{P}_{k\sigma}(E)$ are the self-energy and the force operator, respectively. The presence of the force operator is due to the redistribution of the spectral weight and is an intrinsic feature of SCES. The concept of the force operator was introduced earlier in a diagram technique for spin systems [14]. The Green function $\hat{G}_{k\sigma}^{(0)}(E)$ in Eq. (9) is defined by the formula

$$\left[\hat{G}_{k\sigma}^{(0)}(E)\right]^{-1} = \hat{G}_0^{-1}(E) - \hat{P}_{k\sigma}(E)\hat{T}_{k\sigma}, \quad (10)$$

where $\hat{T}_{k\sigma}$ is the interaction matrix element (for the Hubbard model, $T_{k\sigma}^{m,p} = \gamma_\sigma(m)\gamma_\sigma^*(p)t_k$).

In the Hubbard-I approximation at $U \gg W$, the structure of the exact Green function (9) remains unchanged but the self-energy is equal to zero and the force operator $P_{k\sigma}^{m,p}(E)$ is replaced by $P_{k\sigma}^{m,p}(E) \to P_{0\sigma}^{m,p} = \delta_{m,p}F_\sigma^m$, where $F_\sigma^m \equiv F(p,q) = \left\langle X_f^{pp}\right\rangle + \left\langle X_f^{qq}\right\rangle$ is the occupation factor, which is referred to as the end factor in the diagram technique for the X operators developed in [15]. In terms of the Hubbard-I approximation, the following formula is derived from Eq. (9):

$$\hat{D}_{k\sigma}^{(0)} = \left\{\hat{G}_0^{-1}(E) - \hat{P}_{0\sigma}\hat{T}_{k\sigma}\right\}^{-1}\hat{P}_{0\sigma}. \quad (11)$$

In order to estimate the contributions to Eq. (11) in higher order approximations (with respect to the Hubbard-I approximation), let us compare the exact equation (9) for the Green function with Eq. (11), written in the Hubbard-I approximation. First, there is a difference due to the renormalization of the occupation factors F_σ^m which arises when the exact equation for the force operator $\hat{P}_{k\sigma}(E)$ is used. However, taking into account the corrections due to the force operator keeps the structure of the Hubbard bands unchanged and, therefore, does not lead to a qualitative difference of the exact Green function from that in the Hubbard-I approximation. A second essential difference is the renormalization of the real part of the self-energy $\hat{\Sigma}_{k\sigma}(E)$ and the appearance of quasiparticle damping. The latter implies non-Fermi liquid behavior of the system and, as mentioned above, the consideration of the region where $\mathrm{Im}\Sigma_k(\varepsilon_F) = 0$. In this region, the exact Green function given by Eq. (9) can be rewritten as the sum of one-pole contributions over the quasiparticle bands labeled by index ξ (for the tJ model, ξ has one value, $\xi = 1$; for the Hubbard model, $\xi = 1,2$). In the general multiband case, the exact Green function is

$$G_{k\lambda\sigma}(E) = \sum_\xi \frac{F_{k\lambda\sigma}(\xi)}{E - \Omega_{k\sigma}(\xi) + \mu + i\eta}. \quad (12)$$

Here, the real part of the self-energy contributes not only to the renormalization of the dispersion law but also to the renormalization of the spectral weight. Such a representation for the electron Green function has been obtained earlier in the Hubbard model in terms of the spectral density approach (SDA) [16]. This approach is nonperturbative and assumes only the absence of the quasiparticle damping. The spectral weights $F_{k\lambda\sigma}(\xi)$ and the band energies $\Omega_{k\sigma}(\xi)$ are calculated in the SDA by using the method of moments (see the review and comparison with other methods in [17]).

As for the renormalization of the real part of the self-energy, this effect introduces corrections to the energy spectrum $\Omega_{k\sigma}(\xi)$ and qualitatively does not change the further reasoning. The fact that the structure of the Green function is correct even in the Hubbard-I approximation (it is the structure of the Green function that is essential for further derivation of Luttingers theorem) follows from a comparison of the Hubbard-I solution and the exact solution in the infinite-dimensionality limit obtained in DMFT [4, 5], as well as from a comparison of the Hubbard-I solution and a numerical solution obtained using the exact Quantum Monte Carlo (QMC) method for the Hubbard model [18, 19]. A comparison of the spectral densities obtained in the Hubbard-I approximation at $U \gg W$ and those derived by the QMC shows numerical coincidence between them as in paramagnetic [18] as in antiferromagnetic phases [19]. In terms of the diagram technique for the X-operators, it has also been demonstrated that this approximation gives simple and pictorial relationships which correctly describe the physics of the phenomena at $U \gg t$ [13, 20].

Substituting Eq. (12) into Eq. (7) and using the spectral theorem, we obtain

$$\begin{aligned}
\langle n_{k\lambda\sigma}\rangle &= \int dE\, f_F(E) \sum_\xi F_{k\lambda\sigma}(\xi)\,\delta(E - \Omega_{k\sigma}(\xi) + \mu) \\
&= \sum_\xi F_{k\lambda\sigma}(\xi)\,f_F(\Omega_{k\sigma}(\xi) - \mu), \quad (13)
\end{aligned}$$

Taking into account that the quantities in Eq. (13) do not depend on spin in the paramagnetic phase, the average number of particles $\langle N\rangle$ at zero temperature can be written in the compact form

$$\langle N\rangle = \sum_{k,\lambda}\langle n_{k\lambda\sigma}\rangle = \sum_{k,\xi} F_k(\xi)\,\theta(\mu - \Omega_k(\xi)), \quad (14)$$

where $F_k(\xi) = 2\sum_\lambda F_{k\lambda\sigma}(\xi)$.

For noninteracting particles, we have $F_k(\xi) = 1$ and the equation for $\langle N\rangle$

$$\begin{aligned}
\langle N\rangle &= \sum_k \theta(\mu_0 - \varepsilon_k) = \frac{V}{(2\pi)^3}\int dk\,\theta(\mu_0 - \varepsilon_k) \\
&= \frac{V}{(2\pi)^3}V_{FS}^0, \quad (15)
\end{aligned}$$

completely coincides with Eq. (2) in this case.

For the system of interacting quasiparticles, Eq. (14) can be written as

$$\langle N \rangle = \frac{V}{(2\pi)^3} \sum_\xi \int dk F_k(\xi)\, \theta\left(\mu - \Omega_k(\xi)\right)$$

$$= \frac{V}{(2\pi)^3} \sum_\xi V_{FS}^\xi. \qquad (16)$$

By comparing Eq. (15) with Eq. (16), we obtain

$$V_{FS}^0 = \sum_\xi V_{FS}^\xi, \qquad (17)$$

where V_{FS}^ξ is the volume of the energy subband ξ taking into account the spectral weight $F_k(\xi)$ of this subband,

$$V_{FS}^\xi = \int dk F_k(\xi)\, \theta\left(\mu - \Omega_k(\xi)\right). \qquad (18)$$

Equation (17) is the generalized Luttinger theorem: the right-hand side of the equality is a superposition of the volumes V_{FS}^ξ for the different energy subbands ξ rather than the volume V_{FS} and each state $|k, \sigma\rangle$ for the band ξ enters with a decreased spectral weight.

Therefore, the region bounded by the Fermi surface in the k space becomes "less dense". Indeed, let us use the relationship $m_{FS} = \rho_{FS} V_{FS}$, where m_{FS} is the "mass" of the Fermi surface, ρ_{FS} is its "density", and V_{FS} is its volume. It is obvious that the "mass" m_{FS} is proportional to the average number of particles $\langle N \rangle$ and ρ_{FS} is the spectral weight of the quasiparticles under Fermi surface. For the system of noninteracting particles, we have $m_{FS}^0 = \rho_{FS}^0 V_{FS}^0$. Further, from the equality $m_{FS} = m_{FS}^0$, it follows that $\rho_{FS} V_{FS} = \rho_{FS}^0 V_{FS}^0$ and

$$V_{FS} = \frac{1}{\rho_{FS}} V_{FS}^0, \qquad (19)$$

since for the system without interaction we have $\rho_{FS}^0 = 1$. It is seen that, if the spectral weight of the quasiparticles differs from unity V_{FS} is not equal to V_{FS}^0. It is precisely this case ($\rho_{FS} \leq 1$) that is realized in SCES. On the other hand, the quantity given by Eq. (19) is invariant under the interaction in the system; therefore, the generalization of Luttingers theorem for quasiparticles is as follows: the volume of the Fermi surface of a system of noninteracting particles is equal to that of interacting quasiparticles with account for the spectral weight of these quasiparticles. This formulation of the theorem is valid for both the band electrons and the quasiparticles in metallic SCES in the limit $U \gg W$.

The deviation of the spectral weight from unity can be considered as the transition to a space with a different metric. This is demonstrated in Section V, in which it is shown that the quantity given by Eq. (19) rather than by Eq. (1) is invariant under this transition.

III. T-J MODEL

The Hamiltonian of the Hubbard model in the X-operator representation [11] has the form

$$H = \sum_{f,\sigma} \left((\varepsilon - \mu)\left(X_f^{\sigma\sigma} + X_f^{SS}\right) + \frac{U}{2} X_f^{SS} \right)$$

$$+ \sum_{f \neq g, \sigma} t_{fg} \left(X_f^{\sigma 0} + 2\sigma X_f^{S\bar\sigma}\right)\left(X_g^{0\sigma} + 2\sigma X_g^{\bar\sigma S}\right). \quad (20)$$

The Hamiltonian of the tJ model can be derived form Eq. (20) in the limit of the strong Coulomb interaction $U \gg t$:

$$H_{t-J} = \sum_{f,\sigma} (\varepsilon - \mu)\, X_f^{\sigma\sigma} + \sum_{f \neq g, \sigma} t_{fg} X_f^{\sigma 0} X_g^{0\sigma}$$

$$+ J \sum_{f \neq g} \left(\mathbf{S}_f \mathbf{S}_g - \frac{1}{4} n_f n_g \right), \qquad (21)$$

where t_{fg} is the hopping integral, J is the exchange integral, $\mathbf{S}_f$ is the spin operator, and n_f is the number of particles operator. Since only one fermi-type root vector $\left\{ X_f^{0\sigma} \right\} \leftrightarrow \left\{ X_f^1 \right\}$ is present then the Green function in the case of $\text{Im}\Sigma_k(E) = 0$ has the form

$$D_{k\sigma}(E) = \frac{F(1)}{E - E_k + \mu}, \qquad (22)$$

where $F(1)/2 = (1 - x)/2$ is the spectral weight of the only band $\xi = 1$ and E_k is the spectrum of the system in the Hubbard-I approximation:

$$E_k = \varepsilon + t_k \left(\frac{1 - x}{2} \right) - \frac{J}{2} \left(\frac{1 + x}{2} \right). \qquad (23)$$

Here, $t_k = 2t\left(\cos k_x + \cos k_y\right)$ is the Fourier transform of the hopping integral in the case of a square lattice. The number of particles is

$$\langle N \rangle = \sum_{k,\sigma} \left\langle X_k^{\sigma 0} X_k^{0\sigma} \right\rangle = \sum_k (1 - x) f_F(E_k - \mu). \quad (24)$$

At zero temperature, we have

$$\langle N \rangle = \sum_k (1 - x)\, \theta(\mu - E_k). \qquad (25)$$

This equation coincides with Eq. (14). Thus for the volume of the Fermi surface we have $V_{FS}^0 = F(1) V^- = (1 - x) V^-$, where $V^- = \int dk\, \theta(\mu - E_k)$. In multiband models, such as the Hubbard model, the spectral weight of the quasiparticles is redistributed between the bands due to strong correlations. In t-J model case, there is only one band, but its spectral weight is smaller than unity [see Eq. (25)] because the part of the spectral weight goes to the upper Hubbard band. In deriving the equations for the tJ model from the Hubbard model, this band is taken into account only in terms of the perturbation theory with respect to the parameter $t/U \ll 1$ and does not appear in the Hamiltonian (21) because of the constraint of no two particle excitations $\left\langle X_f^{SS} \right\rangle \to 0$.

IV. HUBBARD MODEL

The basis fermion operators for the Hubbard model (20) are $\left\{ X_f^{0\sigma}, X_f^{\bar{\sigma}S} \right\} \leftrightarrow \left\{ X_f^1, X_f^2 \right\}$, where $|S\rangle$ is a two-particle singlet, $|0\rangle$ is the vacuum state, and $|\sigma\rangle$, $|\bar{\sigma}\rangle$ are one-particle states. The matrix Green function has the form

$$\hat{D}_{k\sigma}(E) = \frac{1}{\det} \begin{pmatrix} \frac{F(1)}{2} \left(E - \varepsilon + \mu - U - t_k \frac{1+x}{2} \right) & 2\sigma t_k \frac{F(2)}{2} \frac{F(1)}{2} \\ 2\sigma t_k \frac{F(1)}{2} \frac{F(2)}{2} & \frac{F(2)}{2} \left(E - \varepsilon + \mu - t_k \frac{1-x}{2} \right) \end{pmatrix}, \tag{26}$$

where $\det = \left(E - E_k^+ + \mu \right) \left(E - E_k^- + \mu \right)$, $F(1)/2 = (1-x)/2$ and $F(2)/2 = (1+x)/2$ are the occupation factors of the lower and upper Hubbard bands, respectively. The energy spectrum of the system is

$$E_k^{\pm} = \frac{1}{2} \left(t_k + U \pm \sqrt{t_k^2 + U^2 + 2t_k U x} \right). \tag{27}$$

The number of particles can be easily found using the Green function (26):

$$\begin{aligned} \langle N \rangle &= \sum_k \left\{ \left[1 + \frac{x(U + t_k x)}{E_k^+ - E_k^-} \right] f_F \left(E_k^+ - \mu \right) \right. \\ &\quad + \left. \left[1 - \frac{x(U + t_k x)}{E_k^+ - E_k^-} \right] f_F \left(E_k^- - \mu \right) \right\}. \end{aligned} \tag{28}$$

This equation coincides with Eq. (14). The expressions in square brackets are the spectral weights of the upper and lower Hubbard bands, respectively. Their sum (taking into account the spin) is equal to the spectral weight of noninteracting electrons, as it must be. Now, let us analyze Eq. (28) in the regime of strong Coulomb repulsion $U \gg t$. In this case, the denominator $E_k^+ - E_k^-$ can be expanded in powers of the small parameter $t/U \ll 1$. Neglecting second-order terms, we have

$$\langle N \rangle = \sum_k \left((1+x) f_F \left(E_k^+ - \mu \right) + (1-x) f_F \left(E_k^- - \mu \right) \right). \tag{29}$$

At zero temperature, this equation becomes

$$\langle N \rangle = \sum_k \left((1+x) \theta \left(\mu - E_k^+ \right) + (1-x) \theta \left(\mu - E_k^- \right) \right). \tag{30}$$

The expression for the volumes of the Fermi surfaces is

$$\begin{aligned} V_{FS}^0 &= F(2) V^+ + F(1) V^- \\ &= (1+x) V^+ + (1-x) V^-. \end{aligned} \tag{31}$$

where $V^{\pm} = \int dk \, \theta \left(\mu - E_k^{\pm} \right)$. It is clearly seen that the due to strong Coulomb interaction the spectral weight is redistributed between the lower and the upper Hubbard subbands. It is this effect that makes generalization of

FIG. 2: Dependence of the chemical potential μ on x.

Luttingers theorem for strongly correlated electron systems is of essence.

The splitting into two Hubbard bands is clearly seen in Eqs. (29), (30) and (31). It is easy to transform the Hubbard model to the tJ model simply by neglecting the influence of the upper (or lower) band, because the bands are separated by a gap U (the interband hopping was already eliminated by the expansion in powers of t/U). The occupation numbers are immediately found to be

$$\lim_{U \to \infty} \langle N \rangle = \sum_k (1-x) f_F \left(E_k^- - \mu \right). \tag{32}$$

This expression coincides with Eq. (24), obtained for the number of particles in the tJ model. Thus, the decrease in the spectral weight in the tJ model is a result of the approximations used ($t/U \ll 1$) rather than of its spontaneous disappearance.

In Figs. 2 and 3 the results of calculations for zero temperature at $U = 10|t|$, $t = -0.2\,eV$ are shown. Calculations for finite temperatures were also carried out, but they did not reveal any qualitative difference from the case of zero temperature. The chemical potential μ calculated self-consistently by using Eq. (28) is shown
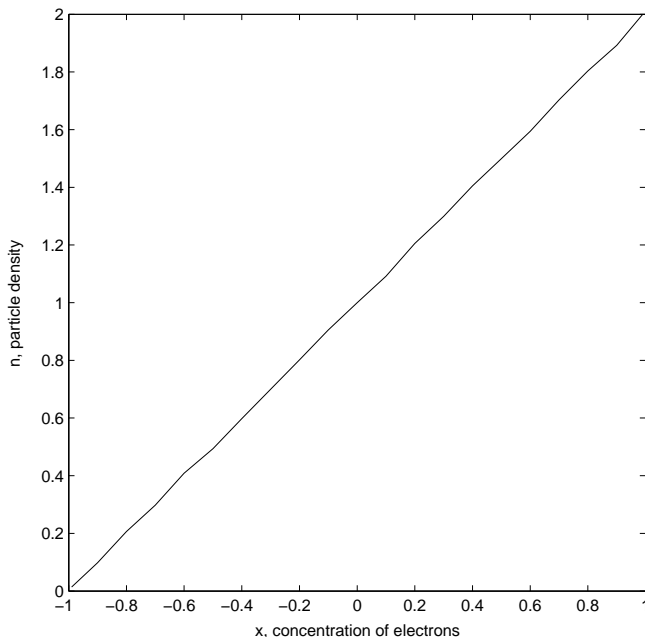
state takes place. Actually, the original Luttinger theorem in the form of Eq. (1) is not valid for SCES; the proof of theorem (1) is not applicable to these systems because it does not take into account the difference of the spectral weight of quasiparticles from unity, which is one of the most remarkable feature of SCES [13]. In present paper we have formulated Luttinger's theorem (17) generalized for the case of quasiparticle description. Qualitative analysis of this generalization given by Eq. (19), as well as analytically exact derivation of Eq. (37) for the scalar densities, showed that the region of k space under the Fermi surface becomes less compact (or, in other words, less "dense") in quasiparticle systems: the contribution of each state is renormalized by decreased spectral weight of the corresponding quasiparticle.

The momentum space is divided into quantum cells, each of which can contain one electron or, taking into account the Pauli principle, two electrons with opposite spins. Some states from the whole set of quantum states in a cell move away to infinite energies due to strong electron correlations. Therefore, the spectral weight F of the remaining states is smaller than unity; quasiparticle excitations in such a system become renormalized, and their spectral weight $F < 1$. It is this effect that causes the k space to be less compact.

It is seen from Eq. (37) for the scalar densities that we deal with a compressible Fermi liquid. A normal Fermi liquid belongs to a subclass of the class of compressible Fermi liquids; in this subclass, the spectral weights of the quasiparticles are equal to unity. The generalized Luttinger's theorem is formulated for the case of a compressible Fermi liquid in which the effects of strong electron correlations necessitate deviation from the description of the system as a normal Fermi liquid.

In this paper we have considered basic models of strongly correlated systems, such as the t-J model and the Hubbard model. It has been shown that in the non-superconducting paramagnetic phase these models satisfy a generalized Luttinger's theorem. In the Hubbard model the spectral weight is redistributed between the Hubbard subbands; in the t-J model a decrease in the spectral weight occurs because part of the states moves away to infinite energies due to the strong correlation between the electrons (the upper Hubbard subband is separated form the lower band by a gap $U \ll t$).

[1] J.M. Luttinger, Phys. Rev. 1194, 1153 (1960)
[2] M. Oshikawa, Phys. Rev. Lett. 84, 3370 (2000)
[3] W. Metzner and D. Vollhardt, Phys. Rev. Lett. 62, 324 (1989)
[4] X.Y. Zhang, M.J. Rozenberg, and G. Kotliar, Phys. Rev. Lett. 70, 1666 (1993)
[5] A. Georges, G. Kotliar, W. Krauth, and M.J. Rozenberg, Rev. Mod. Phys. 68, 13 (1996)
[6] D.M. Edwards and J.A. Hertz, Physica B (Amsterdam) 163, 527 (1990)
[7] Th.A. Maier, Th. Pruschke, and M. Jarrell, Phys. Rev. B 66, 075102 (2002)
[8] Yu.A. Izyumov, Usp. Fiz. Nauk 165, 403 (1995) [Phys. Usp. 38, 385 (1995)]
[9] J. Schmalian, M. Langer, S. Grabowski, and K.H. Bennemann, Phys. Rev. B 54, 4336 (1996)
[10] J. Beenen and D. M. Edwards, Phys. Rev. B 52, 13636 (1995)
[11] J.C. Hubbard, Proc. Roy. Soc. London A 276, 238 (1963)
[12] J.C. Hubbard, Proc. Roy. Soc. London A 277, 237 (1964)

[13] V.V. Valkov and S.G. Ovchinnikov, Quasiparticles in Strongly Correlated Systems (Sib. Otd. Ross. Akad. Nauk, Novosibirsk, 2001)
[14] V.G. Baryakhtar, V.N. Krivoruchko, and D.A. Yablonskiǐ, Greens Functions in the Theory of Magnetism (Naukova Dumka, Kiev, 1984)
[15] R.O. Zaǐtsev, Zh. Eksp. Teor. Fiz. 70, 1100 (1976) [Sov. Phys. JETP 43, 574 (1976)]
[16] W. Nolting, Z. Phys. B: Condens. Matter 255, 25 (1972)
[17] T. Herrmann and W. Nolting, J. Magn. Magn. Mater. 170, 253 (1997)
[18] C. Grober, R. Eder, and W. Hanke, Phys. Rev. B 62, 4336 (2000)
[19] S.G. Ovchinnikov and E.I. Shneyder, Central Eur. J. Phys. 3, 421 (2003)
[20] Yu.A. Izyumov and Yu.N. Skryabin, Basic Models in the Quantum Theory of Magnetism (Ural. Otd. Ross. Akad. Nauk, Yekaterinburg, 2002)